Virtual Polywell - take 3

Brute Force Electron Tracking

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Introduction

In the previous versions of the Virtual Polywell the fundamental assumption has been that the electron distribution just "appeared" at time zero. The idea was to follow something from a pre-existing state or to have it be created by microwave ionization. Creating a self consistent distribution based on Maxwellian thermal velocity distributions failed to work.

In this version I start from a vacuum condition and use electron sources to initialize the system. This is exactly how a real Polywell is initialized, so in one sense this attempt is more realistic. However, real electron sources have real velocity distributions, and even though they are small, it is unlikely this model will be able to account for that level of detail. Instead, the electrons will be assumed to be born with a delta function velocity and each blob of electrons will be followed as a single particle until it either hits a wall or MaGrid coil.

The MaGrid magnetic field has been previously described in the first "Virtual Polywell" paper (fusion.pdf). This model will continue to use that magnetic field form. The electric field is also described, although not in as much detail, in the same paper. These external fields to the system are fixed and for this model will not change.

An electron source is a ball of copper or some other material which can be placed in any position outside the MaGrid. There are two main positions worth modeling, one with a source on the face of a coil, and one in a corner between 3 coils - a cusp. Bussard's WB-6 experiment had electron sources in the cusp position. In either case, this model will assume symmetry all around the "wiffleball" with 6 sources on a face or 8 sources on cusps.

The main line of attack for this article will be to derive the potential around the electron sources. The method will be to transform to a 2D problem from symmetry, and then use a mapping to a simpler 1D problem so that the potential can be easily found. A method to use the mapping and the derivative of this potential will give us the electric field due to the electron sources everywhere in the Polywell volume.

Each electron source will be divided up into a set of surfaces. Each surface will represent one delta function particle. A simple formula will be derived so that the number of particles can be easily changed and the areas on the electron source sphere will remain uniform. At any given time step, these particles will be given an energy equivelent to the work function of the surface so that they leave the electron source with very small velocity. The rest will be pure brute force particle tracking.

The equations of the electric and magnetic fields created by the particles will be derived based on the fact that each particle will have the same amount of charge. This makes the computations tractable, and we should be able to follow up to 1000 time steps with little trouble.

Derivation of Electric Potential from Electron Sources

The general description of the electron source is very simple. It is a solid sphere which we can take to be "magically" suspended at some location inside the grounded sphere of the vacuum chamber. In reality there will be some kind of insulated holder with a conductor in the middle of it, but for an analytical treatment we can ignore this minor interruption to the potential. I will also assume that half the sphere is insulated and the half exposed towards the center of the Polywell is emmitting electrons. The main reason for this is to suppress current loss directly to ground.

Call the diameter of the grounded outside sphere is labled s, the diameter of the electron source is labled r_e, and the location of the electron source relative to the center of the sphere is labled r_s. Eventually we will take these as parameters relative to the MaGrid distance between coil face and center of the Polywell which will have a real distance L. For our formulation, this distance will be 1 so that scaling will be easy.

Since everything is a sphere, cutting a plane through the centers of both spheres s and r_e will give us a two dimensional problem which is rotationally symmetric. If the sphere r_e is held at some potential V_e relative to the external ground, we can find the potential every where in the volume by solving a two dimensional boundary value problem

$$\nabla^2\Phi = 0 \tag{1}$$

with conditions

$$\Phi(r_e) = V_e \tag{2}$$

and

$$\Phi(s) = 0 \tag{3}$$

This problem is actually fairly messy, so an alternative is to use conformal mapping to reduce this to a one dimensional problem. The mapping will take the circle r_e located off center by r_s to be centered inside the circle s. To do this, we take polar coordinates r, φ in the base domain (centered on s) and transform them to r_w, θ centered on r_e in the solution domain.

The result is that the outside ground is like a cam rather than the inside sphere being a small range in φ. If we write the form of the outside ground in Cartesian coordinates and set both forms equal in both x and y we get

$$r_s + r_z\cos\theta = s\cos\varphi \tag{4}$$

$$r_z\sin\theta = s\sin\varphi \tag{5}$$

where r_w is the distance from the center of the electron source to the outside ground. With these two equations we can eliminate r_w and find φ in terms of θ. Substitution of

$$r_z = \frac{\sin\varphi}{\sin\theta}s \tag{6}$$

from (5) into (4) gives

$$(s\cos\varphi - r_s)\tan\theta = s\sin\varphi \tag{7}$$

Since $\sin\varphi = \sqrt{1-\cos^2\varphi}$ we can solve this nonlinear equation directly by squaring and using the quadratic formula. The result after a bit of algebra is

$$\cos\varphi = \frac{r_s\sin^2\theta \pm \cos\theta\sqrt{s^2 - r_s^2\sin^2\theta}}{s} \tag{8}$$

Putting this back into (4) and solving for r_w we find

$$r_z = -r_s\cos\theta + \sqrt{s^2 - r_s^2\sin^2\theta} \tag{9}$$

To do the complete transform we need to map any point between the electron source and the outside ground using the angle θ. The electron source surface itself has its center at the origin of θ, but the outside ground has its center at $y = -r_s$ from the electron source. A reasonable mapping will be one that can deal with both of these surfaces and transform any point in between as well.

Placing a circle with its origin some where between the outside ground origin and the electron source origin and using this as a mapping surface allows us to transform the region between the two surfaces in very simple way. Take the radius of the circle to be

$$s(t) = r_e + t(s - r_e) \tag{10}$$

and the origin of the circle to be

$$r_s(t) = tr_s \tag{11}$$

where $0 < t < 1$ so that when $t = 0$ we have $s(0) = r_e$ and $r_s(0) = 0$ which maps the inside circle (electron source) and when $t = 1$ we have $s(1) = s$ and $r_s(1) = r_s$ which maps the outside ground. Putting (10) and (11) into (9) in place of s and r_s gives

$$r_z(t) = -tr_s \cos\theta + \sqrt{(r_e + t(s - r_e))^2 - t^2 r_s^2 \sin^2\theta} \tag{12}$$

In the transformed domain the angle stays the same, but the distance from the center is now a radius:

$$r_w(t) = r_e + (s - r_e)t \tag{13}$$

We now have a system which is independent of angle - the location between the spheres is given by the distance from the center of the electron source and all angles are the same. Solving Poissons equation is now trival even in spherical coordinates since only the derivative is radial and all angular dependence has been removed.

The basic form of the equation is given in (1) and in cylindrical coordinates we are left with

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) = 0 \tag{14}$$

Since $r = 0$ is not even part of the solution volume, we directly integrate this and get

$$\Phi(r) = \int \frac{K}{r^2} dr = -\frac{K}{r} + C \tag{15}$$

The boundary conditions (2) and (3) give us values for K and C with

$$V_e = -\frac{K}{r_e} + C \tag{16}$$

and

$$0 = -\frac{K}{s} + C \tag{17}$$

Solving for K and C and putting this into (15) we find

$$\Phi(r_w) = \frac{V_e r_e}{s - r_e}\left(\frac{s}{r_w} - 1\right) \tag{18}$$

which is valid in the transformed domain. To use this, we need to go from a known position (r, φ) in the original problem to (r_z, θ) in the shifted domain to the (r_w, θ) in the solution domain. The value of $\Phi(r_w)$ is then the same value of the potential we need for the point (r, φ). We can also find the electic field at any point due to the electron source by computing the gradiant of (18). This gives

$$\vec{E} = \frac{\partial \Phi}{\partial r_w} = -\frac{V_e r_e}{s - r_e} \frac{s}{r_w^2} \hat{r}_w \tag{19}$$

In other words, the electric field points towards the center of the electron source as if it were a point particle. Since this is a conformal mapping, the angles between tangents stay the same, so the electric field in the original domain also has the electric field pointing into the center of the electron source.

If we pick an arbitrary point in the region between the electron source and the ground and call this (r, φ) which is referenced to the center of the Polywell, then we can convert to Cartesian coordinates using

$$x_b = r \sin \varphi \tag{20}$$

$$y_b = r \cos \varphi \tag{21}$$

(since φ is referenced to the y axis).

Calling (r_z, θ) the same point in the referenced to the electron source this same Cartesian point is

$$x_d = r_z \sin \theta \tag{22}$$

$$y_d = r_z \cos \theta + r_s \tag{23}$$

Setting (22) equal to (20) and (23) equal to (21) we find

$$r_z = \sqrt{(y_b - r_s)^2 + x_b^2} \tag{24}$$

and

$$r_z \cos \theta = y_b - r_s \tag{25}$$

Solving (12) for t allows us to to find the location of r_w using (13). The result is

$$t = \frac{r_e(s - r_e) - r_s r_z \cos\theta + \sqrt{(r_s r_z \cos\theta - r_e(s - r_e))^2 - \left(r_s^2 - (s - r_e)^2\right)\left(r_z^2 - r_e^2\right)}}{r_s^2 - (s - r_e)^2} \tag{26}$$

So the process of finding the electric field due to an electron source (or the potential) is to use (20) and (21) to convert to Cartesian form, use (24) and (25) to find the terms used in (26) which gives the transform parameter t that can be used in (13) to give us the point in the solution domain. This value of r_w then can be used in (19) (or (18)) to give the Electric field (or potential).

Initial Delta Function Phase Space

From "Classical Electrodynamics" by Jackson (pg 56) we have a relationship between the surface charge and electric field given by

$$\frac{\partial \Phi}{\partial r} = -\frac{\sigma}{\varepsilon_0} \tag{27}$$

Setting this equal to equation (19) at $r_w = r_e$ we get

$$\sigma = \frac{\varepsilon_0 V_e s}{r_e(s - r_e)} \tag{28}$$

which is the total charge per unit area. The total surface area of the sphere is $4\pi r_e^2$ so the total charge on the sphere is

$$Q = \frac{4\pi\varepsilon_0 V_e s r_e}{(s - r_e)} \tag{29}$$

Since I only want half the sphere to be emmitting electrons, the total charge leaving at any instant in time has to be half of (29). Since the charge distribution (28) is independent of angle, we can divide the surface of the electron source into equal segments, each containing the same amount of charge. By giving each of these blobs of charge an initial energy of just over the work function of the surface, we can follow every blob as a delta function particle in phase space assuming that its initial velocity is radially directed away from the center of the electron source sphere.

One of the major advantages of the 6 sided cube form of the Polywell is the amount of symmetry. As Indrek showed before, there are 48 copies of the same corner which can be folded around the faces with 6 sub sections in each of 8 main sections created by mid-plane cuts. Putting an electron source on a cube face means we have 8 copies of each source section rotated around but putting an electron source at a cusp gives us 6 copies of each source section (with 8 total cusp points for electron sources).

This symmetry greatly reduces the number of particles we need to follow so long as we can assume all the fields stay symmetrical. Reality is never so nice, but it should give us some good clues as to what kinds of problems we'll see and solutions we can try to overcome them. So let's suppose we can divide these 6 or 8 slice sections of electron source into evenly divided areas. What will they look like?

First, let's create a coordinate system and call ψ the angle from the axis pointing towards the center of the Polywell on the electron source and χ the angle from one symmetry section face. Assume we can chop up the section into even areas. The emmision solid angle of a face electron source will be

$$\Delta\psi\Delta\chi = \frac{2\pi}{8N} \tag{30}$$

and the emission from a cusp will be

$$\Delta\psi\Delta\chi = \frac{2\pi}{6N} \tag{31}$$

Where N is the total number of particles we are going to follow.

The centroid of each chunk of solid angle can be found by brute force from

$$\psi_c = \frac{\int_{\psi_1}^{\psi_2} \psi \sin\psi d\psi}{\int_{\psi_1}^{\psi_2} \sin\psi d\psi} \tag{32}$$

and

$$\chi_c = \frac{\int_{\chi_1}^{\chi_2} \chi d\chi}{\int_{\chi_1}^{\chi_2} d\chi} \tag{33}$$

These can be directly integrated and we get

$$\psi_c = \frac{\sin\psi_2 - \sin\psi_1 + \psi_1\cos\psi_1 - \psi_2\cos\psi_2}{\cos\psi_1 - \cos\psi_2} \tag{34}$$

and

$$\chi_c = \frac{\chi_2 + \chi_1}{2} \tag{35}$$

It is clear we can divide the bands across each segment into equal sections no matter how the bands are separated in ψ. Let's take 1 segment at the apex with 2 segments below that and 3 below that and so on until we get to the equator. For m bands we must have a total number of segments equal to

$$N = \frac{m(m+1)}{2} \tag{36}$$

Across each band j, there are j segments of χ. Since the total angle across a band is either $\frac{\pi}{4}$ on a face or $\frac{\pi}{3}$ on a cusp the width of each χ segment must be

$$\Delta\chi = f\frac{\pi}{j} \tag{37}$$

where $f = 1/4$ on a face and $f = 1/3$ on a cusp.

depending on where the electron source is located. Taking the first χ line to be zero, the next one over is $f\frac{\pi}{j}$ and the k^{th} one over will be $f\frac{\pi k}{j}$. If we start counting from 0, we can write the centroid of any χ segment k as

$$\chi_c = f\frac{\pi}{2j}(2k+1) \tag{38}$$

Putting (37) into (30)/(31) we can eliminate $\Delta\chi$ and get

$$\Delta\psi = \frac{j+1}{N} \tag{39}$$

From the denominator of the ψ centroid we know what $\Delta\psi$ is so we must have

$$\frac{j+1}{N} = \cos\psi_j - \cos\psi_{j+1} \tag{40}$$

From (36) we have a recursive formula for the angular spread in ψ which is

$$\cos\psi_{j+1} = \cos\psi_j - \frac{2(j+1)}{m(m+1)} \tag{41}$$

It is straight forward to derive from the known end points of $\cos\psi_0 = 1$ and $\cos\psi_{m+1} = 0$ that (41) becomes

$$\cos\psi_{j+1} = 1 - \frac{j(j+1)}{m(m+1)} \tag{42}$$

Choosing values for m and j this is easy to verify. The trick here is that j ranges from 0 to m but our $\Delta\chi$ starts at $j = 1$ because division by 0 is not useful (in equation (37)). The software has to keep track of which row we're really on.

Computationally we can now find the centroid of any segment on the electron source - use (42) to find the cut, compute the inverse cosine of this to get ψ_j, and then the sine of ψ_j for each band so that the centroids can be computed using (34).

The next task is to convert these centroids from electron source coordinates into the 3D coordinates of the Polywell system.

Cusp and Face Position and Velocity Vectors

The next step is to place the electron source in the Polywell coordinate system. For computational purposes I take the postion in spherical coordiantes (r, θ, φ) with θ measured up from the (x, y) plane instead of down from the z axis as is normally done. The velocity vector I keep in Cartesian coordinates so that the calculation of vector additions doesn't go haywire in the center of the Polywell. The conversion between electron source coordinates and Polywell coordinates is performed using Cartesian coordinates as a go between, so the vector direction of the source particles becomes trivial.

Placing the electron source on the x axis with its "top" pointing toward the center of the Polywell and its χ angle measured up from the (x, y) plane I can write the orientation of the electron source as

$$\begin{aligned} \hat{x}_e &= -\hat{x} \\ \hat{y}_e &= -\hat{y} \\ \hat{z}_e &= \hat{z} \end{aligned} \tag{43}$$

with each of the above being a unit vector along it's respective axis. The position of a source point in the electron source coordinate system is

$$x' = r_e \cos\psi \tag{44}$$

$$y' = -r_e \sin\psi \cos\chi \tag{45}$$

$$z' = r_e \sin\psi \sin\chi \tag{46}$$

The translation from the electron source coordinate system to the Polywell coordinate system is given by

$$\vec{r} = (r_s - x')\hat{x} - y'\hat{y} + z'\hat{z} \tag{47}$$

The radial distance from the center of the Polywell is given by

$$r^2 = (r_s - x')^2 + y'^2 + z'^2$$

or after a little algebra

$$r^2 = r_e^2 + r_s^2 - 2 r_e r_s \cos\psi \tag{48}$$

The angle φ in the Polywell coordinate system is

$$\varphi = \tan^{-1}\frac{y}{x} \tag{49}$$

From (44), (45) and (47) we see this is

$$\varphi = \tan^{-1}\left(\frac{r_e \sin\psi \cos\chi}{r_s - r_e \cos\psi}\right) \tag{50}$$

The angle θ in the Polywell system (for our calculations anyway) is given by

$$\theta = \tan^{-1}\frac{z}{\sqrt{x^2 + y^2}} = \tan^{-1}\frac{z}{\sqrt{r^2 - z^2}} \tag{51}$$

Using (48) and (46) the last form is simple to derive and we get

$$\theta = \tan^{-1}\left(\frac{r_e \sin\psi \sin\chi}{\sqrt{r_e^2 + r_s^2 - 2r_e r_s \cos\psi - r_e^2 \sin^2\psi \sin^2\chi}}\right) \tag{52}$$

Thus, for any centroid position (ψ_c, χ_c) on the electron source, we can find this position in the Polywell reference frame for the source placed on the coil face position.

The direction the particle initially starts out with will be radially outward from the center of the electron source. This vector is simply

$$\vec{u} = \vec{r} - \vec{r}_s = -r_e \cos\psi\, \hat{x} + r_e \sin\psi \cos\chi\, \hat{y} + r_e \sin\psi \sin\chi\, \hat{z} \tag{53}$$

Now let's look at the electron source in the cusp location. Here, I let the "top" of the electron source point toward the center of the Polywell along the cusp axis. Placing the $\hat{x}_e$ axis in the same plane as the $\hat{z}$ and $\hat{z}_e$ axies, the $\hat{y}_e$ axis is parallel to the (x, y) plane. The associated transformation vectors of the center of the electron source is then found to be

$$\hat{x}_e = \frac{\hat{x}}{\sqrt{6}} + \frac{\hat{y}}{\sqrt{6}} - \frac{\sqrt{2}}{\sqrt{3}}\hat{z} \tag{54}$$

$$\hat{y}_e = \frac{\hat{x}}{\sqrt{2}} - \frac{\hat{y}}{\sqrt{2}} \tag{55}$$

$$\hat{z}_e = -\left(\frac{\hat{x}}{\sqrt{3}} + \frac{\hat{y}}{\sqrt{3}} + \frac{\hat{z}}{\sqrt{3}}\right) \tag{56}$$

As before we can take the coordinate transformation as

$$\vec{r} = \vec{r}_s + \vec{r}_e \tag{57}$$

With the same form of (44) and (46) but the sign is now switched on (45) since we've accounted for this in (55). The direction vector $\vec{r}$ in Cartesion form is now given by the three terms

$$\hat{x}\text{:}\quad \frac{r_e \sin\psi \cos\chi}{\sqrt{6}} + \frac{r_e \sin\psi \sin\chi}{\sqrt{2}} - \frac{r_e \cos\psi}{\sqrt{3}} + \frac{r_s}{\sqrt{3}} \tag{58}$$

$$\hat{y}\text{:}\quad \frac{r_e \sin\psi \cos\chi}{\sqrt{6}} - \frac{r_e \sin\psi \sin\chi}{\sqrt{2}} - \frac{r_e \cos\psi}{\sqrt{3}} + \frac{r_s}{\sqrt{3}} \tag{59}$$

$$\hat{z}\text{:}\quad -\frac{\sqrt{2}\, r_e \sin\psi \cos\chi}{\sqrt{3}} - \frac{r_e \cos\psi}{\sqrt{3}} + \frac{r_s}{\sqrt{3}} \tag{60}$$

This distance r from center of Polywell to the source point is exactly the same as before (which you can verify by squaring (58), (59) and (60) and adding them all up) to be the same as equation (48). Using (49) with (58) and (59) we find the angle from the x axis in the (x, y) plane to be

$$\varphi = \tan^{-1}\left(\frac{r_e\left[\sin\psi\left(\cos\chi - \sqrt{3}\sin\chi\right) - \sqrt{2}\cos\psi\right] + \sqrt{2}r_s}{r_e\left[\sin\psi\left(\cos\chi + \sqrt{3}\sin\chi\right) - \sqrt{2}\cos\psi\right] + \sqrt{2}r_s}\right) \tag{61}$$

From (51), (60) and (48) we find

$$\theta = \tan^{-1}\left(\frac{r_s - r_e\left(\sqrt{2}\sin\psi\cos\chi + \cos\psi\right)}{\sqrt{2}\left\{r_e^2\left(1 + \frac{1}{2}\sin^2\psi - \sin\psi\cos\chi\left(\sin\psi\cos\chi + \sqrt{2}\cos\psi\right)\right) + r_s^2 - r_s r_e\left[2\cos\psi - \sqrt{2}\sin\psi\cos\chi\right]\right\}^{1/2}}\right) \tag{62}$$

Using the same form as (53) we can see that the direction vector leaving the surface of the electron source located on the cusp is given by

$$\vec{u}(\text{cusp}) =$$

$$\hat{x}\colon\ \frac{r_e}{\sqrt{6}}\left(\sin\psi\left(\cos\chi + \sqrt{3}\sin\chi\right) - \sqrt{2}\cos\psi\right) \tag{63}$$

$$\hat{y}\colon\ \frac{r_e}{\sqrt{6}}\left(\sin\psi\left(\cos\chi - \sqrt{3}\sin\chi\right) - \sqrt{2}\cos\psi\right) \tag{64}$$

$$\hat{z}\colon\ -\frac{r_e}{\sqrt{6}}\left(\sqrt{2}\sin\psi\cos\chi - \cos\psi\right) \tag{65}$$

Particle Equations of Motion

We are now ready to start working on a very simple model of electron tracking in the Polywell configuration. Using an electron source at either the coil face or cusp location and using the $\frac{1}{48}$ symmetry of the Polywell, we can follow a few particles released at every time step using the location and vector direction described above. The amount of charge is determined by the voltage on the electron source as shown in (29) and this is divided by the total number of blobs to be followed, with each blob containing an equal amount of charge.

This latter approximation is very useful for speeding up computer calculations. It allows us to give both position and velocity as delta functions as well as the amount of charge being fixed. The electric field (in SI units) for the particles is given by

$$\vec{E}_f = \frac{1}{4\pi\varepsilon_0}\sum_i \rho_i \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \tag{66}$$

and for the magnetic field generated by these particles moving is given by

$$\vec{B}_f = \frac{\mu_0}{4\pi}\sum_i \vec{J}(\vec{r}_i) \times \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \tag{67}$$

Converting to the same kind of dimensionless form used in previous versions these become

$$\vec{\mathcal{E}}_f = \frac{1}{4\pi\varepsilon_0 LV}\sum_i \rho_i \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \tag{68}$$

$$\vec{\mathcal{B}}_f = \frac{1}{L I_0}\sum_i \vec{J}(\vec{r}_i) \times \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \tag{69}$$

To convert (29) to dimensionless form, let's take

$$V_e = p_e V \tag{70}$$

and include a factor f as in (37) to account for either a face or cusp location. If we also divide by the factor of 2 since only half the sphere is assumed emmitting particles, the total charge per "blob" is then

$$Q_i = \frac{\pi\varepsilon_0 VL}{N} f \frac{p_e s r_e}{(s - r_e)} \tag{71}$$

Putting (71) into (68) and noting that $\vec{J} = \rho\vec{u}$ in (69) we find

$$\vec{\mathcal{E}}_f(\vec{r}) = \frac{p_e s f r_e}{2N(s - r_e)} \sum_i n_i(\vec{r}) \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \tag{72}$$

$$\vec{\mathcal{B}}_f(\vec{r}) = \pi\varepsilon_0 \frac{V v_0}{2I_0} \frac{p_e s f r_e}{(s - r_e)} \sum_i n_i(\vec{r}) \vec{\mathrm{u}}(\vec{r}_i) \times \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \tag{73}$$

Combining the electric field from the electron source along with the MaGrid electric and magnetic fields shown in previous work along with the above forms of the particle fields, we can compute the electric and magnetic fields for every particle in the Polywell. Using these fields along with the Lorentz force equation gives a complete model of particle motion. To convert the force equation to dimensionless form we start with

$$\vec{F} = Q_i \left[\vec{E} + \vec{E}_f + \vec{v}_i \times \left(\vec{B} + \vec{B}_f \right) \right] \tag{74}$$

Since we are assuming the mass of each particle does not change, we can take $\vec{F}$ to be

$$\vec{F} = m \frac{v_0 \Delta \vec{\mathrm{u}}}{\Delta t} \tag{75}$$

where we are taking the acceleration as the change in velocity between time steps. The charge per blob is given in (71). Using the same conversions between (66) $\rightarrow$ (68) and (67) $\rightarrow$ (69) along with (71) and (75) we can write equation (74) as

$$m \frac{v_0 \Delta \vec{\mathrm{u}}}{\Delta t} = \frac{\pi\varepsilon_0 VL}{N} f \frac{p_e s r_e}{(s - r_e)} \left[\frac{V}{L} \left(\vec{\mathcal{E}} + \vec{\mathcal{E}}_f \right) + v_0 \frac{\mu_0 I_0}{4\pi L} \vec{\mathrm{u}} \times \left(\vec{\mathcal{B}} + \vec{\mathcal{B}}_f \right) \right] \tag{76}$$

After a little rearranging we can turn (76) into

$$\Delta \vec{\mathrm{u}} = \frac{f p_e s r_e}{2N(s - r_e)} \left[\vec{\mathcal{E}} + \vec{\mathcal{E}}_f + \frac{\vec{\mathrm{u}} \times \left(\vec{\mathcal{B}} + \vec{\mathcal{B}}_f \right)}{\mathcal{C}_p} \right] \Delta\nu \tag{77}$$

where I have used the same dimensionless parameters from previous papers, namely

$$\mathcal{C}_p = \frac{2\pi}{\mu_0 I_0} \sqrt{\frac{2mV}{e}} \tag{78}$$

and

$$\nu = \frac{e\mu_0 I_0}{4\pi m L} t \tag{79}$$

Summary

The order of operations for a computer model will be to first set up dimensionless parameters for p_e, s, r_e, r_s and $\mathcal{C}_p$. We can start by using reasonable values for L, V, I_0 and V_e to see where this starts us, then try alternative values for the dimensionless parameters to see what happens and work backwards to find out where that puts us for actually building something. In any case, the static fields for the MaGrid only need be computed once and the field produced by the electron sources (19) only needs to be computed once, then stored in a data table for the remainder of the program run.

The changing fields and currents due to the free moving particles is modeled by (72) and (73). All the fields are then used in (77) to determine how far every particle goes in one time step. The change in position is computed from the change in velocity using

$$\vec{\mathfrak{u}}_i(\nu + \Delta\nu) = \vec{\mathfrak{u}}_i(\nu) + \Delta\vec{\mathfrak{u}}_i(\nu) \tag{80}$$

and

$$\vec{\mathfrak{r}}_i(\nu + \Delta\nu) = \vec{\mathfrak{r}}_i(\nu) + \Delta\nu\, \vec{\mathfrak{u}}_i(\nu + \Delta\nu) \tag{81}$$

At every time step we can allow (or disallow) particles leaving the electron source surface. The initial velocity should be close to the work function of the surface so that it is very small, but finite and definitely leaving on a specific trajectory. The ability to control the number of particles so that we can limit either the time it takes to do a calculation or simply to limit the pulse width of the injection current gives us a lot of flexability and learning opportunities for this model.

Next step is to see what happens with simple and straight foward code.