THE QUANTUM MECHANICAL PATH INTEGRAL: TOWARD A REALISTIC INTERPRETATION

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ABSTRACT

In this paper, I explore the feasibility of a *realistic* interpretation of the quantum mechanical path integral — that is, an interpretation according to which the particle actually follows the paths that contribute to the integral. I argue that an interpretation of this sort requires spacetime to have a branching structure similar to the structures of the branching spacetimes proposed by previous authors. I point out one possible way to construct branching spacetimes of the required sort, and I ask whether the resulting interpretation of quantum mechanics is empirically testable.

1. The Path Integral: A Philosophical Perspective

In the path integral formulation of quantum mechanics [Feynman and Hibbs 1965], the transition amplitude between two quantum states of a system is expressed as a sum over contributions from possible classical histories of that system. The mathematical object used to represent this sum is an integral known as a "path integral." It is important to note that the histories used in this integral are *classical* histories. For example, if the system is a particle, then the path integral runs over classical trajectories of the particle. These trajectories are "classical" only in the sense that they are sharp curves; they need not satisfy the laws of classical mechanics, and may wander through space in ways that Newton's laws would not allow. Nevertheless, a particle following a single one of these trajectories would have only one position at each time, just as a classical particle would. In the simplest example of a path integral, the transition amplitude for the particle to travel from point **r** at time t to point **r**' at time t' is expressed as a certain integral over all of these classical trajectories that take the particle from (**r**, t) to (**r**', t'). For obvious reasons, the path integral approach to quantum mechanics often is called the "sum over histories" approach.

From a physicist's standpoint, the most important fact about the path integral formalism is that it works. The formalism can be used to obtain predictions of the behavior of physical systems; these predictions are found to agree with experiment and with the predictions of other formulations of quantum mechanics. The question of whether a particle described by the path integral *really* traverses all the trajectories in the integral is a question that some physicists might view with impatience as meaningless, or at least as irrelevant to physics. However, from a philosophical standpoint, it may be interesting to ask what the path integral really represents. Are the trajectories used in the path integral real? Is there really anything that traverses those trajectories when the particle travels from here to there? Or is the path integral simply a formal device, useful for calculation, but not describing any real motions along classical paths? Such questions should not be regarded as pointless; they are nothing less than special cases of the important and widely studied philosophical problem of realism vs. antirealism with respect to theoretical entities.

The question of the reality of the paths in the path integral may or may not be directly relevant to the practice of physics. However, it certainly is relevant to our understanding of physics. Physicists routinely use a number of ideas that do not actually contribute to the computational success of physical theories, but that nevertheless are useful because they give us a mental picture of what physical theories describe. The widely accepted Copenhagen interpretation of quantum mechanics is like this. An interpretation of the quantum mechanical path integral — a mental picture of what the path integral represents — might be helpful for understanding path integral quantum mechanics in the same way as the Copenhagen interpretation is useful for understanding the physical meaning of wave mechanics or Hilbert space.

In this note, I wish to explore one particular interpretation of the path integral: what one might call the *realistic* interpretation. This is the view that takes the path integral at face value, and supposes that the paths used in the integral are objectively real — in other words, that the particle actually follows these paths. Prima facie, this view seems outrageous: it appears to imply (for example) that a particle actually takes more than one classical path through a single space, and hence exists wholly in more than one place at a single time. My first task will be to show that the realistic interpretation does not actually have this strange consequence. I will point out a way to implement the realistic interpretation of the path integral without postulating a space full of classical paths, and without positing any physical entities that actually occupy more than one position on a given spacelike slice of spacetime.

It is likely that some readers will find the realistic interpretation of the path integral to be intuitively unacceptable from the outset. However, this interpretation will turn out to lack much of the oddness that it initially seems to have. In any case, the realistic interpretation is of interest because many people's thinking about path integrals is implicitly based on this interpretation. Those of us who use path integrals also tend to use the mental picture of sums over multiple classical histories — although perhaps few of us fully believe this picture. It is possible, and I think common, to visualize path integrals as sums over histories without taking this identification too literally — that is, without really believing that the system actually pursues each classical path available to it. In studying the realistic interpretation of the path integral, we can perhaps find out whether any sense can be made of the alternative view that takes the sum over histories literally.

Throughout this note, I will focus on the path integral for a single nonrelativistic point particle. When I say "path integral," I will usually have this particular path integral in mind. I do this for the sake of simplicity; most of my remarks will apply, with certain reservations, to more general path integrals.

2. The Case Against Multiple Paths

The path integral, if interpreted in a naive and literal-minded way, seems to tell us that a particle can follow many different classical trajectories at once. The principal reason that this conclusion is hard to believe is the prima facie conflict between this conclusion and widely accepted scientific views of space and time. According to one generally accepted belief about physical reality, it is impossible for a physical object to be entirely in one place at one time and also be entirely in another place at the same time. If a particle actually followed each of a set of distinct classical trajectories through the same space at the same time, then this basic assumption about physical reality would be violated. There is a fairly widespread feeling that quantum mechanics has undermined the view that a particle has only one position at a time. Nevertheless, when we think of the delocalized character of objects in quantum mechanics, we do not picture this in terms of an object being definitely here, and also definitely there. Instead, we think of an object as being here with a certain probability, and there with a certain probability — with no fact of the matter as to where the object "really is." This kind of thinking is consistent in spirit with the Copenhagen interpretation. We think of a particle that is not in an eigenstate of position as an object without a definite position — not as an object with multiple definite positions, each of which it fully occupies. Thus, the idea that a particle follows more than one classical trajectory simultaneously is somewhat foreign to our customary thinking about quantum mechanics, as well as being alien to classical mechanics. This is the case even though we use this idea as an intuitive picture while constructing path integrals.

If we tried to interpret the sum over histories in a way reminiscent of the Copenhagen interpretation, then we might come up with an interpretation like this: for each path, there is a certain probability for the particle to follow that path, but there is no fact of the matter as to which path the particle really follows. However, if we interpret the path integral realistically, then we arrive at a slightly different view: that the particle really follows each path, and that each path really contributes in some way to the outcome of the transition.

Let us next find out whether this literal-minded view — in which the particle really does follow multiple paths — can be forced to make sense.

3. Spacetime and Sums Over Histories

The chief argument against the realistic interpretation of the path integral is the apparent conflict between this interpretation and certain accepted ideas about spacetime. To better understand the nature of this conflict, we should examine our conception of spacetime more closely.

Normally, we think of spacetime as a manifold. In general relativity, spacetime has the structure of a differentiable manifold, and almost always that of a Hausdorff differentiable manifold. In a spacetime with this latter structure, the trajectory of a classical point particle is a single curve. For each value of the time (at least as measured in an ordinary, well-behaved coordinate system), there is one and only one space point at which the particle is located. If for some particle this were not the case — if at some time, the particle existed in two or more places — then we would be inclined to speak of there being *two or more* particles instead of *one* particle. Instead of getting into questions of identity and individuation of particles, I will simply take it for granted here that a classical particle in a Hausdorff differentiable manifold can have only one position at a time.

The picture begins to look a bit different if we allow spacetime to be a differentiable manifold with a *non-Hausdorff* structure. Most of the spacetime manifolds familiar to us are Hausdorff manifolds. However, non-Hausdorff manifolds have long been known to physics. Non-Hausdorff manifolds play a role in general relativity, specifically in the theory of Taub-NUT space (see [Hawking and Ellis 1973]). In certain other non-Hausdorff manifolds (see [Visser 1995] and [Penrose 1979]), it is possible for a particle to pursue a continuous trajectory and nevertheless reach two different events which are assigned the same time by the particle's clock. There is no magic in these manifolds; they just have a spacetime topology different from what we are used to. Their topology gives them a "branching" structure that is reminiscent of the branching of history in the many-worlds interpretation of quantum mechanics. Indeed, some authors have studied the question of whether the many-worlds interpretation (at least in some of its variants) might require us to assume a non-Hausdorff structure for spacetime ([Penrose 1979], [Visser 1995]). Branching and non-Hausdorff spacetimes have been used in the interpretation of quantum mechanics (for example, in [Belnap 2003], [McCall 1995], and [Douglas 1995]) and in the philosophical analysis of indeterminism (see especially [Belnap 2003], [Belnap 1998], [McCall 1995], and [Douglas 1995]).

Before proceeding, I must say a few words about the scientific credibility of the concept of branching spacetime, and of non-Hausdorff spacetime in particular. As I have pointed out, non-Hausdorff spacetimes are nothing new to physics (recall Taub-NUT space). The other kind of non-Hausdorff spacetime described above, in which the trajectory of a single particle can undergo branching, is more speculative. However, several authors have taken the idea of branching spacetime seriously enough to investigate its possible uses. Visser

[1995] has suggested that non-Hausdorff branching spacetime might be one way to defuse the consistency difficulties that arise in chronology violating spacetimes. (This latter idea was explored again in [Sharlow 1998].) Penrose [1979] and Visser [1995] have remarked on the possible relevance of branching spacetime to the interpretation of quantum mechanics. Belnap (in, for example, [Belnap 2003]) has used non-Hausdorff branching spacetime in the interpretation of quantum mechanics and in the study of physical indeterminism. Douglas [1995] and McCall [1995] have employed branching models of spacetime in the interpretation of quantum mechanics and in the study of the problem of the direction of time. (Penrose [1979] also has related branching spacetime to this latter problem.) In view of all this, it would be silly for physicists to regard the idea of branching spacetime as unacceptably speculative — especially in view of the situation in string theory, where empirically unsupported speculations are routinely taken quite seriously. In any case, most applications of branching spacetime to quantum mechanics are philosophical interpretations rather than empirical hypotheses, so evaluating these interpretations is a task for philosophy and not for physics.

The main real objection to branching spacetime is a philosophical one based on the threat of endless multiplication of objects or observers due to the branching of trajectories [Penrose 1979]. I think that this objection can be answered successfully by one or more of the following means: (1) arguments showing that the multiplication would be unobservable, similar to arguments deployed in defense of many-worlds quantum mechanics ([Everett 1957], [DeWitt 1970]); (2) topological arguments suggesting that the multiplication need not compromise the identities of observers [Sharlow 1998]; (3) some mechanism like the "branch attrition" discussed by McCall [1995]. Because of the relatively large number of potential ways out of this objection, I will not consider this objection further in this paper.

The realistic interpretation of the path integral appears to push us in the direction of a non-Hausdorff spacetime topology. To see why this is, we will next analyze in some detail the following familiar scenario: A particle is placed in a state of definite position **r** at time t, then moves freely thereafter. From quantum mechanics, we know that at any time t' later than t, the particle will be in a state that is not an eigenstate of position; in other words, the particle becomes spatially delocalized. For simplicity in studying this scenario, we will assume that the spacetime in which the particle moves is geometrically flat, and we will use only inertial coordinate systems.

Consider the paths that the particle in our scenario takes between t and t'. All of these paths are continuous curves, and all of them begin at the same event E — the event of the release of the particle. Each of these paths reaches a constant-time hypersurface at t', and when any path does so, it reaches that hypersurface at some space point, which may be different for different paths. Hence the paths begin at a single event E, and branch apart somewhere between t and t'. However, if you are an observer, and you search for the particle over all the space accessible to you at t' (that is, a maximal connected spacelike hypersurface), you will not find multiple particles. Instead, you will find only *one* particle in all of space. This is the outcome that quantum mechanics predicts. *Yet this outcome can happen only in a branching spacetime.* If the particle's trajectory has multiple future branches, and at any given time t' each of those branches reaches a different maximal connected spacelike slice at t', then the spacetime must branch.

This argument suggests that branching manifolds might provide models of spacetime in which a particle can consistently follow many classical histories at once. The idea that the sum-over-histories picture of quantum mechanics might push us toward a branching view of spacetime is not at all new (see Visser [1995] and Penrose [1979]). As I mentioned earlier, various authors have proposed that quantum mechanics itself can be interpreted with the help of branching spacetime.

Once we admit the possibility that spacetime might branch, the idea of a single particle pursuing many single-valued trajectories begins to look quite natural. In a branching spacetime, it is easy for a particle to follow multiple trajectories without ever giving rise to multiple particles that can be observed at the same time.

4. A Sample Non-Hausdorff Spacetime

If we want to interpret the path integral in terms of non-Hausdorff spacetime, we must find a non-Hausdorff spacetime structure that actually reproduces the result of the path integral. At minimum, we need to find a non-Hausdorff manifold in which: (1) a classical point particle has a history consisting of many different trajectories, like the sum over paths used in the path integral, and (2) the transition amplitude for a particle to travel from one coordinate point to another is the same as what one gets from the customary path integral.

It is not hard to satisfy part (1) of this requirement. A spacetime structure that will do this is shown in Figure 1. (I am not claiming that spacetimes like the one in Figure 1 are the only ones potentially useful for interpreting the path integral. The spacetime of Figure 1 is an example, meant to show that it is logically possible to construct a spacetime that satisfies (1).) Note that the spacetime in Figure 1 is either the same as or closely related to a branching spacetime depicted by Penrose [1979], and probably to those described by Belnap [2003], McCall [1995], and Douglas [1995] as well. The ideas that I am about to present here have many points of contact with the ideas of all of these authors, and also of Visser [1995].

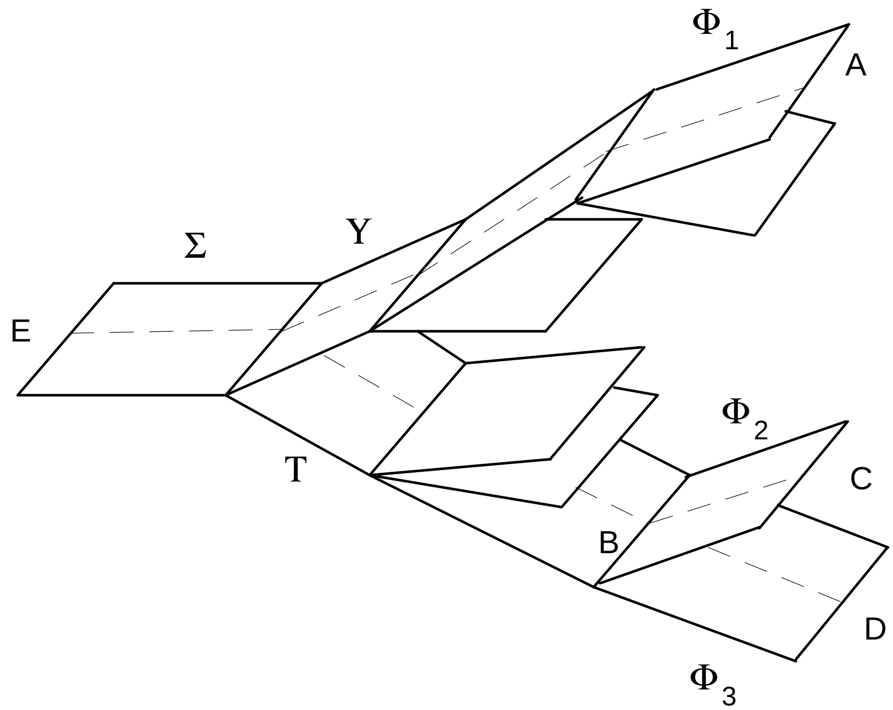

Figure 1.

The spacetime shown in Figure 1 consists of several "sheets," or maximal Hausdorff submanifolds not containing any hypersurfaces at which branching occurs. These sheets are glued together at their boundaries in such a way that any given boundary is the future end of one sheet and the past end of more than one other sheet. We may assume each sheet to be a Lorentzian manifold with boundary; for simplicity, we will take the sheets to be geometrically flat. We will leave the shape of the boundaries undecided, although light cones form a plausible choice (see especially [Visser 1995] and [Penrose 1979]). In Figure 1, the boundaries are shown as straight lines for graphical clarity.

Now let us return to the scenario of the previous section. Consider what happens if we release a classical particle at event E in the spacetime of Figure 1. Beginning at E, the particle pursues multiple trajectories through the spacetime, traversing different sheets as it goes. Three of these many trajectories are shown in Figure 1 as dotted lines (EA, EBC and EBD). When a trajectory hits a boundary, that trajectory branches out into all sheets on the future side of the boundary. However, at most one curve arising from the trajectory crosses each sheet. What happens when an observer detects the particle? The event of detection must occur in one of the sheets — or, if the process of detection takes long enough, in a series of temporally consecutive sheets. Hence the observer will detect only *one* particle, even though there are many branches in the particle's trajectory.

The net result is that the particle released at E makes its way through spacetime along many different curves. If one prefers not to speak of a particle as having more than one trajectory, then one might want to say instead that different copies of the particle follow different trajectories. No matter how one chooses to describe it, the physics comes out the same. (By this last remark, I do not mean to suggest that this difference in terminology is philosophically insignificant.)

It is important to notice that we can apply much of the apparatus of spacetime physics to a branching spacetime like the one in Figure 1. (Some such apparatus already has been used in previous authors' work on branching spacetime models.) We can do this because the spacetime consists of pieces that are ordinary Hausdorff spacetime manifolds. On any given sheet of a branching spacetime, we can define coordinate patches, geodesics, and all the rest of the apparatus of the differential geometry of manifolds. We even can define these over parts of the spacetime much larger than a single sheet. If we pick out a particular sheet, and trace forward to successive future sheets while selecting only one future sheet at each boundary, then we get a series of sheets that form, in effect, an ordinary non-branching spacetime manifold. The chain of sheets from E to A in Figure 1 is a series of this sort. From the standpoint of an observer in a sheet in such a chain, there is a unique past spacetime geometry that appears to be Hausdorff. This geometry even can be extended to include the entire future, provided a single forward chain of branches is chosen. The resulting "spacetime," which actually is a subspace of the non-Hausdorff spacetime, may be called a *superbranch*. It is possible to define coordinate systems, etc. on a superbranch. A superbranch is much the same as, and possibly identical to, what Belnap [2003] calls a "history," and to what Douglas [1995] calls a "reality." In this paper

I have used the word "history" differently, to refer to the history of a physical object or observer.

Consider the particle whose trajectories are shown in Figure 1. What does this particle's history look like to an observer? The observer's history, like the particle's history, will undergo branching. Because the observer's history is branched, there will be many possible outcomes when the observer tries to measure the particle's position or other properties. For example, suppose that the observer releases the particle at E, then waits until a later time t' as measured on a clock comoving with the observer. The observer follows various paths, including three particular trajectories that enter the sheets labeled Φ_1, Φ_2 and Φ_3. Suppose that on each of these three paths, the observer's clock reads t' when the observer is somewhere in the sheet Φ_1, Φ_2 or Φ_3. Then any measurement that the observer makes at t' will reflect the state of the particle in either Φ_1, Φ_2 or Φ_3. Thus, the outcome of a measurement depends, not only upon the paths taken by the particle, but also upon the paths taken by the observer who makes the measurement. This last statement is, I think, is generally true for any branching model of spacetime.

To be of any physical interest, our spacetime model must reproduce the quantum mechanical result that the particle becomes spatially delocalized after t. One might ask whether this can happen at all in the spacetime of Figure 1, under our simplifying assumption that each sheet is flat. If particle and observer both pursue deterministic courses with identical initial conditions through identical (flat) spacetime geometries, then the results of measuring the particle's position should not depend on which branches the particle and observer have followed. The particle and the observer are, in effect, each just following many identical copies of the same trajectory. So how can the particle become delocalized?

The answer to this question depends on a certain detail of the spacetime model. Think about what would happen if we tried to construct the spacetime in Figure 1 from a set of separate sheets, as a topologist might do. To join two sheets together (or to "glue" them together, as mathematicians sometimes say), we must attach the future edge of one sheet to the past edge of the other sheet. For example, we must glue the future edge of Σ to the past edge of T. When we do this, we must decide which points in the future boundary of Σ will be identified with (or co-located with) each point in the past boundary of T. For example, we might stipulate that a particular point P in Σ is to be joined to a point Q in T. However, this mapping between the future edge of Σ and the past edge of T does not completely determine the physical characteristics of the boundary between Σ and T. Consider an inertial test particle in Σ. This particle moves on a curve in Σ. When that particle reaches the point P and crosses from Σ into T, the particle will have to continue its motion on one of the curves that start at Q. The mere fact that the particle reaches the boundary at P and thus reaches Q does not tell us which curve the particle will take after it passes Q. Thus, to completely specify the structure of the spacetime, we must also decide on a correspondence between the curves in Σ which end at each point on the future boundary of Σ, and the curves in T which begin at the corresponding point on the past boundary of T. Once we have done this for every point on the boundary, we have specified the structure of the boundary sufficiently to determine the path of every inertial test particle that crosses the boundary.

By correctly "pairing off" curves in this way in temporally successive sheets, we can ensure that two trajectories originating from the same event end up separating from each other spatially. When a particle and an observer pass through a number of sheets together, the particle may end up displaced relative to its expected position in the observer's frame. Thus, we can construct a spacetime in which different trajectories of the particle end up at different places in an observer's frame. We even can insure that particles which have the same initial 4-momentum will end up with different 4-momenta after they cross a boundary. This phenomenon might seem to violate conservation of 4-momentum, but I do not think it needs to lead to any observable violation; the changes in 4-momentum might average out over many sheets, or might cancel each other out on each single sheet. In any case, this pairing of curves is not the only way to construct a spacetime that can displace the position of a particle at a sheet boundary. Another way is to glue together the edges of the sheets carefully, so that trajectories that hit a boundary at a relative separation **d** from each other have a relative separation different from **d** on the future side of the boundary. In this case, particles undergo an effective jump in position at the boundary. I will leave the details of this construction to the topologically inclined reader.

Both of these constructions introduce an effective deviation from flatness into the spacetime geometry. Presumably this can be represented mathematically as some kind of curvature. However, in both cases, the deviation is *restricted to the boundaries between sheets*. The interiors of the sheets remain flat. Hence the set of points with nonzero curvature is a set of measure zero in the spacetime. "Curvature" of this sort should not be confused with gravitational curvature, which is a different feature of spacetime.

Once we have allowed the geometry of the spacetime to vary at the boundaries in one or both of these ways, we have arrived at a spacetime structure in which delocalization of a particle can occur. Consider our scenario again. The particle is released at E. Later, at t', the observer measures the position of the particle. What the observer finds can vary, depending upon which branch the particle is in when the act of measurement is performed on it. At coordinate time t', the particle and the observer may be found in many different sheets — and the observer, whatever sheet he/she is in at the time, may find the particle to be in any one of many different places on the sheet. This agrees qualitatively with what quantum mechanics predicts. Further, if the observer tries to measure some other property of the particle (like momentum), there also can be a spread of possible values, at least if the property in question can be altered by the crossing of a boundary.

One might object that this picture of spacetime does not accurately reflect the sum-over-histories approach, for the following reason. In the sum-over-histories picture, the final state is the outcome of the particle's traversal of many possible histories. In our present model, the observer ultimately sees the outcome of only *one* possible history of the particle. This seems very different. However, on closer inspection, it is not really all that different. In the model discussed here, the different paths do not actually come back together to create the final state, as they do in our usual intuitive picture of a sum over histories. However, all of the different histories of the particle nevertheless contribute to the outcome of the experiment. The observer has a chance of finding himself in any of a large set of sheets, and therefore has a chance of seeing any one of a large set of trajectories of the particle. The fact that there is a non-vanishing probability for finding any one of these trajectories is a consequence of the fact that the particle follows many different trajectories. Therefore, *all* of the trajectories of the particle contribute at least indirectly to the outcome of the observation, because they contribute to the probabilities for finding the particle at various places in space.

5. Recovering the Path Integral?

Now I will look at part (2) of the task described above. I will ask whether transition amplitudes consistent with the path integral for a single particle can be recovered from the spacetime model just described. I will not actually present a derivation of the path integral from the spacetime model. However, I will point out a direction in which such a derivation might be found.

First, consider the structure of a single particle trajectory on a superbranch. The trajectory extending from E to A in Figure 1 is an example; the chain of sheets extending from Σ to Φ_1 is a superbranch. If we look only at a single superbranch, and ignore all other sheets in the spacetime, then we find that the superbranch is essentially just an ordinary Hausdorff spacetime, cut across by occasional hypersurfaces which may have strange internal geometries. A particle propagating through such a spacetime will have a trajectory that one would expect of a particle in an ordinary spacetime — except that the particle's motion will be perturbed discontinuously now and then, as the particle crosses the special hypersurfaces. (For all we know, this discontinuous change in the state of the particle's motion might not have to result in any radiation, gravitational or otherwise, since the particle's motion within each sheet of spacetime is unaccelerated. After all, the change that occurs at a boundary is not due to real forces or even to gravitational curvature, but simply to the way that the successive sheets are glued together. Also, we cannot rule out the possibility that the distorted geometry on the hypersurfaces will somehow absorb energy from particles, just as the gravitational field can do.) Thus, the kinematics of a single particle will not be conventional classical kinematics, but *stochastic mechanics*.

The use of stochastic mechanics in the study of the foundations of quantum mechanics is not a new idea. It is known that stochastic motion can, under certain conditions, be described by an amplitude $\psi(\mathbf{x}, t)$ that is formally identical to a quantum mechanical wave function ([Nelson 1985], [Nagasawa 1993]). This amplitude obeys a Schrödinger equation, and satisfies the usual probability interpretation with $\psi^*(\mathbf{x}, t)\psi(\mathbf{x}, t)$ equal to the probability density for finding a particle at point $\mathbf{x}$ at time t. There is a widespread feeling that stochastic motion cannot replicate the interference behaviors of quantum wave functions, as typified by the double-slit experiment. Actually, this feeling is not very well-founded; there is a known way to get around this impossibility (see note 1). I have not yet specified the character of the jumps in particle position and/or momentum that occur at the boundaries; I have only pointed out that there are many possible choices for these jumps. So far, there are essentially no constraints on these jumps. We are free to choose the jumps in almost any way we want. Thus, it seems very likely that if we choose the right kind of jumps, we can make the particles in the superbranch undergo a kind of stochastic motion that could satisfy the conditions for the function $\psi(\mathbf{x}, t)$ to exist. All we would have to do is glue the sheets together in such a way as to replicate the jumps prescribed by the theories of stochastic mechanics that give a correct Schrödinger wavefunction. We have not proved that this is possible, but in view of the extreme freedom that we have in choosing the jumps, it is likely to be possible. If we choose the jumps the right way, then the motion of the particle in the superbranch might appear to observers to be describable by wave mechanics. Of course, this could be the case only if the times between jumps (that is, between the boundary hypersurfaces), as measured by the observers, were much smaller than the observers' time resolutions. Otherwise, the motion would simply appear to be a classical stochastic motion. We will call an observer *coarse* if that observer's time resolution is too poor to allow the observer to notice the jumps. For observers with finer time resolutions, the granularity of the spacetime would become evident.

Note that this "wave function" $\psi(\mathbf{x}, t)$ is not just a description of the motion of one trajectory of the particle. Although there is only one trajectory that lies entirely on a given superbranch, various other trajectories overlap that superbranch by sharing initial segments with it. Any particle trajectory contains pieces of other trajectories as initial segments, so $\psi(\mathbf{x}, t)$ summarizes parts of the particle's motion on many different trajectories.

If the motion of a particle really were like this, then a coarse observer who records observations of the particle's past history would be able to describe that particle's history as the evolution of a wave function in accord with a Schrödinger equation. The evolution of this wave function would be deterministic. However, the particle's actual motion would not have to be deterministic, and in any case would not be observable to the coarse observer. This latter motion would be indeterministic if the geometry at each boundary hypersurface were not predictable from the geometry of spacetime in that hypersurface's past. Further, the particle would exhibit behaviors that the coarse observer could interpret as collapse of the wave function. For example, if the observer finds the particle to be at a particular position $\mathbf{r}$, then the observer is in a sheet in which the particle has position $\mathbf{r}$ — so the particle's subsequent motion, as seen by that observer, would have to be consistent with the initial position $\mathbf{r}$. A similar apparent collapse might well occur with the measurement of momentum or other dynamical variables. In the case of these properties, an observer who finds a value a of an observable A could only be on one of the sheets in which A has the value a — but *which* sheet is a matter of chance. The particle's subsequent evolution would be consistent with the initial condition that A has value a. The outcome of this apparent collapse would be truly indeterministic if the underlying particle motion were indeterministic.

In the spacetime model suggested here, a nonrelativistic test particle might well appear to coarse observers to be a quantum mechanical particle. We have not proven that this would be the case, but we have shown that it is a plausible conjecture. If this conjecture turns out to be right, then we would be able to recover the path integral for a point particle indirectly, through the equivalence of part of the path integral formalism with Schrödinger wave mechanics. It might even be possible to derive the path integral, with its complex exponential weighting factor, in a direct manner (not via a Schrödinger equation) from the spacetime model — though we do not presently know how to do this.

In this note, I have concentrated on one very simple special case of the path integral: that of a single nonrelativistic particle. It remains to be seen whether more complicated cases of the path integral also can be interpreted in terms of branching spacetime topologies, though it seems plausible that they can. We can ask this question even for quantum gravity and string theory, where the existence of integrals over different geometries and topologies makes an interpretation in terms of branching manifolds seem especially natural.

6. Is This Interpretation New?

The interpretation of the path integral that I developed in the last section is very similar to several previously known interpretations of quantum mechanics. One of these is the many-worlds interpretation [DeWitt and Graham, 1973]. According to the many-worlds interpretation, the state vector of the universe undergoes a kind of branching that results in multiple histories, much like those shown in Figure 1. The difference between my suggestion and the many-worlds interpretation is twofold. First, the many-worlds interpretation only involves a branching of the histories of quantum systems. That interpretation does not explicitly postulate a branching of the underlying spacetime, although some authors (cited earlier) have asked whether it might lead us to adopt a branching spacetime topology. Second, in my interpretation of the path integral, the quantum state of the particle is not fundamental. Instead, the wave function describes the motion of a particle whose path is classically definite within any given sheet of spacetime. One could hardly call such a particle "classical," since the particle has multiple trajectories and exists in many different histories "at once." The particle that we have been discussing is a quantum mechanical entity; however, the wave function (or state vector) is not the most fundamental level of description of the particle's state.

The interpretation of the path integral that I am proposing here is similar to earlier interpretations that make explicit use of branching spacetime. Interpretations of this kind have been suggested by Belnap [2003], McCall [1995] and Douglas [1995]. My proposed interpretation of the path integral actually is a branching spacetime interpretation of quantum mechanics, and is quite similar to these other interpretations in many respects. The main difference is that in my interpretation, the quantum wave function is derived from an underlying stochastic dynamics.

A third interpretation of quantum mechanics that resembles mine is the stochastic interpretation. I have alluded to that interpretation several times already. The stochastic interpretation is not as widely discussed today as are the Copenhagen and many-worlds interpretations, but it has a long history extending back to the time of Schrödinger (see [Jammer 1974]). There is a large literature on the stochastic interpretation (see [Nelson 1985], [Nagasawa 1993] and [Jammer 1974] for entry points). According to the stochastic interpretation, the formal similarities between quantum mechanics and stochastic mechanics should be taken seriously, and quantum mechanics should be understood as a kind of stochastic dynamics. I will not try to assess the stochastic interpretation here; I will only say that I am deeply indebted to it, and have borrowed from it my ideas about how to model wave functions using stochastic dynamics. My proposed interpretation of the path integral is essentially a hybrid of the stochastic interpretation and the branching spacetime interpretation. It is a stochastic interpretation in non-Hausdorff spacetime. (Its truth does not depend on the truth of the standard stochastic interpretation.)

7. But Is It Really an Interpretation?

This new interpretation of the path integral raises a delicate epistemological question that I have not yet addressed. This is the question of whether the "interpretation" really is a philosophical interpretation at all. Although I set out to find a philosophical interpretation of the path integral, the model at which I finally arrived involves a spacetime structure that looks as if it might be subject to empirical testing. Is the idea presented in this paper really a philosophical interpretation of the path integral, or is it an empirical hypothesis?

My answer to this question is that the idea may indeed be an empirical hypothesis. The spacetime model proposed here implicitly contains a scale of time (call it τ) representing the typical time interval between the boundaries at which sheets meet. If real spacetime were like this model, and if one could probe the behavior of matter over time intervals comparable to τ or smaller, then one would find that there are no quantum fluctuations over those intervals, and that particles appear to one's probes to be spatially localized. Thus, the model predicts that at scales smaller than τ, the behavior of physical systems is not described accurately by quantum mechanics. This consequence of the model may seem strange, but it does not conflict with anything we know today, provided that τ is far below the resolution of today's experiments. If the model is true, then the scale τ must be far below the resolution of any observer presently known to us; otherwise, we would already have encountered the effects just mentioned.

This argument does not prove that our spacetime model is testable. For all we know, there might be some physical reason why we never could make the observations necessary to confirm this prediction. In any case, one could argue that the model is not falsifiable unless we set bounds on the value of τ. If we do not add to the model the stipulation that τ is within a certain range, then every time an experiment fails to find the predicted effects, we can simply say that the resolution of the experiment was not fine enough. However, if we assume a lower bound on τ, and if it is physically possible to probe matter at that scale, then it is physically possible to test the spacetime model.

The model's empirical character need not count against the model as an interpretation of quantum mechanics. Some of the known interpretations of quantum mechanics are empirically testable in principle. I have in mind especially the stochastic interpretation. Although such ideas are not purely philosophical, they still can be considered to be interpretations of quantum mechanics. The peculiar prediction made by our model even could have a scientific advantage, for the following reason. Suppose that we were to generalize our model to incorporate gravitational curvature of spacetime. In such a curved branching spacetime, if τ is of the order of the Planck time (about 5×10^{-44} seconds) or larger, then the breakdown of spacetime structure at the Planck scale due to quantum gravitational fluctuations [Misner, Thorne and Wheeler 1973] would not occur. The model would have a kind of built-in cutoff for most quantum fluctuations. Below τ, the geometry of spacetime would become free of quantum fluctuations and would be smooth, except perhaps on the boundary hypersurfaces, where all of the geometric messiness would be concentrated.

If τ really is as small as the Planck time, then we are very far, technologically speaking, from being able to test the hypothesis that spacetime is like the model described in this paper. The Planck time is far below the time resolutions of experiments available to us today. But practical impossibility is not the same as physical impossibility, and the fact that a test appears to be physically possible suggests that the model has empirical content.

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Notes

1. At first glance, a classically sharp stochastic trajectory would seem incapable of replicating quantum interference effects such as those found in the double slit experiment. However, there does not appear to be any conclusive reason to believe in this impossibility. Nelson [1985] suggested that the double slit experiment has a natural interpretation in stochastic mechanics if a suitable background field drives the stochastic motion. Nelson pointed out that the background field might give rise to correlations between happenings at the two slits, thereby allowing the motion of electrons through slit A to correlate with the motion of other electrons through slit B, and causing the observed interference of the wave function. As I have pointed out elsewhere [Sharlow 2004], the sheet boundaries in branching spacetime might have the same effect as a background field, thereby rendering actual background fields unnecessary. Readers unsure of the relevance of stochastic motion to the interpretation of quantum mechanics may consult the large literature on the stochastic interpretation of quantum mechanics.

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